Title: Analysis of negative index materials in FEKO

Application Note Details

Application Note Number	TA-Metamaterials-07-06
Application	Metamaterials
Special FEKO cards	
Associated files	NIM_metamaterials_in_FEKO.dpc;
Date	31 July 2006

Problem Description

The computational analysis of meta-material structures by the assignment of negative relative permittivity and/or permeability properties to regions in CEM models raises various questions about the accuracy and applicability of different techniques as well as the best modelling approaches in terms of meshing strategy, etc.

Objectives

Confirmation of FEKO's ability to handle negative index materials effectively and the outlining of some basic guidelines for computations based on comparison of results generated using different solution techniques for a simple problem. A demonstration of FEKO's application to the analysis of a real problem.

References

1. "Periodic FDTD Characterization of Guiding and Radiation Properties of Negative Refractive Index Transmission Line Metamaterials", ACES 2006, Costas D. Sarris et al

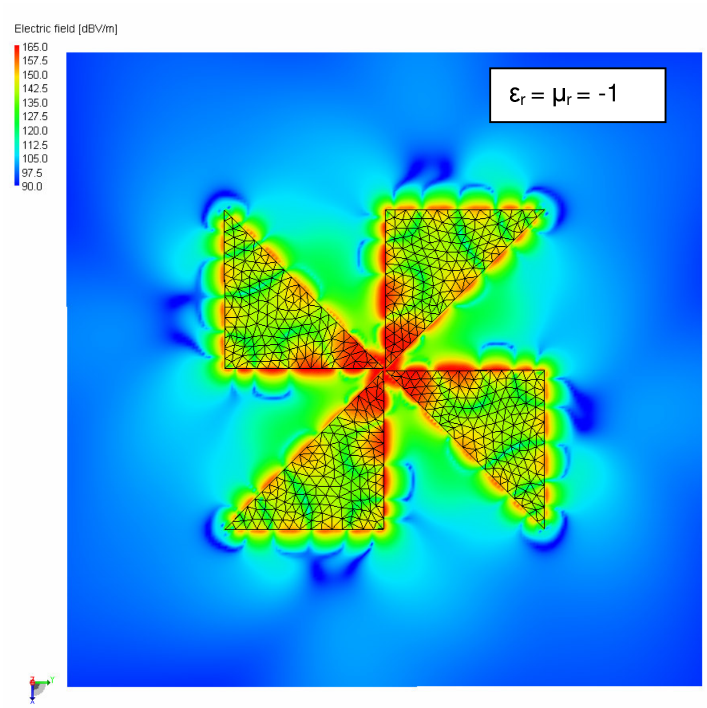


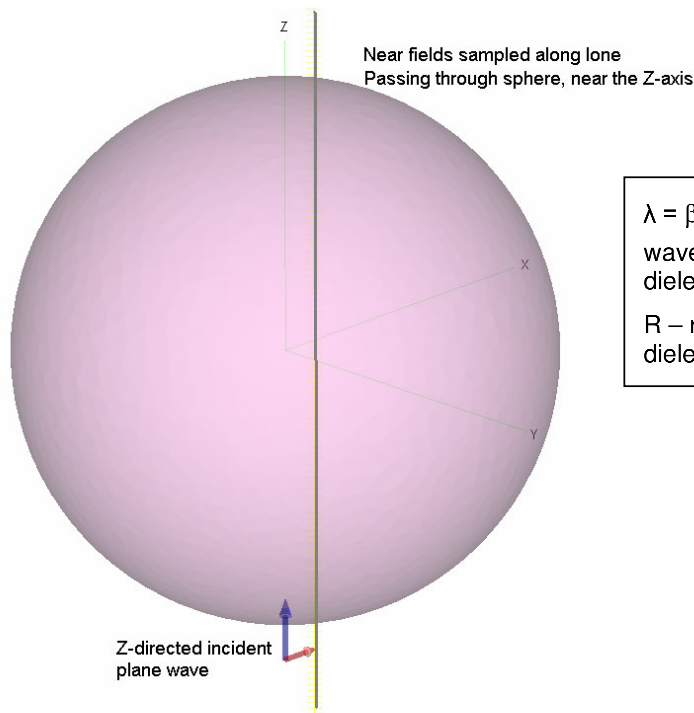
Figure 1. ***Magnitude of the electric near-field in a meta-material resonator central plane at 4.545 GHz***

Fields in an illuminated sphere of negative index material

Technical description

A simple canonical sphere structure with variable radius and dielectric parameters (a problem that can be solved analytically using special Green's function or other series expansion techniques).

The sphere is excited by an incident plane wave in the z-direction and the electric field sampled along a line passing near the centre of the sphere (to avoid analytical singularities) is considered.



$\lambda = \beta r = 1{,}2/\sqrt{\varepsilon_r \mu_r}$ - wavelength in the dielectric material

R – radius of the dielectric sphere

Figure 2. ***Structure of the canonical problem used for defining meshing rules and comparing techniques***

4 cases are considered using 3 different techniques (SEP, VEP and FEM/MoM) in FEKO.

Case 1: Electrically small sphere with low index values (R = $\lambda/4$ with ε_r and μ_r = +/- 1.5)

Case 2: Medium sphere with low index values (R = 2λ with ε_r and μ_r = +/- 1.5)

Case 3: Electrically large sphere with low index values (R = 8λ with ε_r and μ_r = +/- 1.5)

Case 4: Electrically small sphere with high index values (R = $\lambda/4$ with ε_r and μ_r = +/- 8)

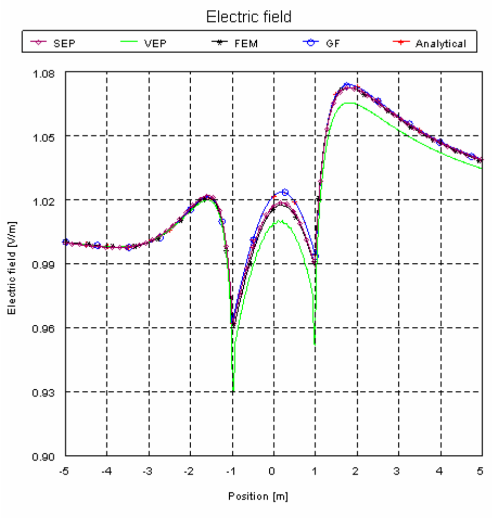
From various simulations using these 4 cases a comparison between solution techniques and some basic guidelines for the implementation of negative index materials for problems of different electrical size in FEKO are extracted. Some selected simulation results are given in the following figures.

Simulation results

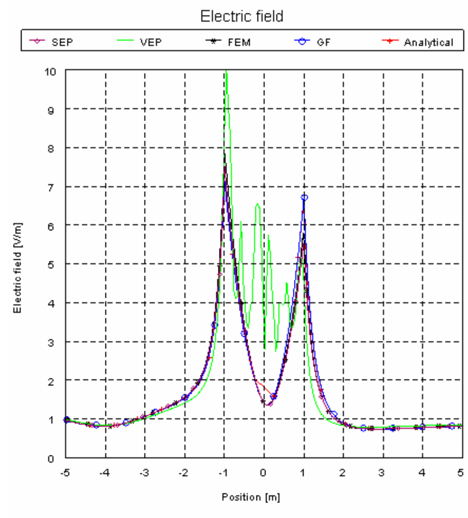
Figure 3 shows simulation results, optimal meshing details and resource requirements for case 1.

ε_r=+1,5 | ε_r = -1,5

μ_r=+1,5

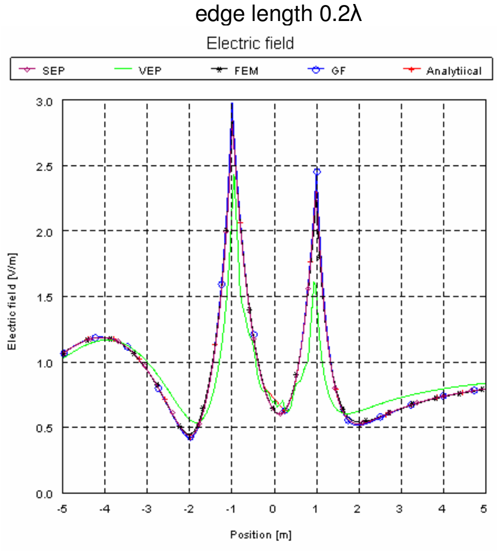


SEP: 1 hour (8 CPU), 2.1 GByte
edge length 0.05λ
VEP: 4 min (8 CPU), 2.2 GByte
edge length 0.1λ
FEM: 16 sec (1 CPU),150 MByte
edge length 0.2λ

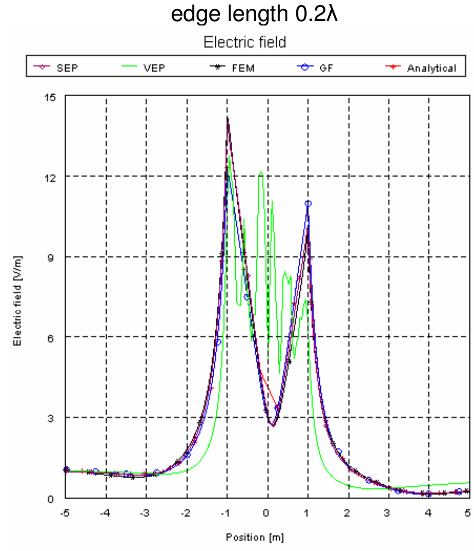


SEP: 16 sec (1 CPU), 50 MByte
edge length 0.2λ
VEP: 6 sec (1 CPU), 40 MByte
edge length 0.2λ
FEM: 16 sec (1 CPU), 150 MByte
edge length 0.2λ

μ_r=-1,5



SEP: 16 sec (1 CPU), 50 MByte
edge length 0.2λ
VEP: 6 sec (1 CPU), 40 MByte
edge length 0.2λ
FEM: 16 sec (1 CPU), 150 MByte
edge length 0.2λ



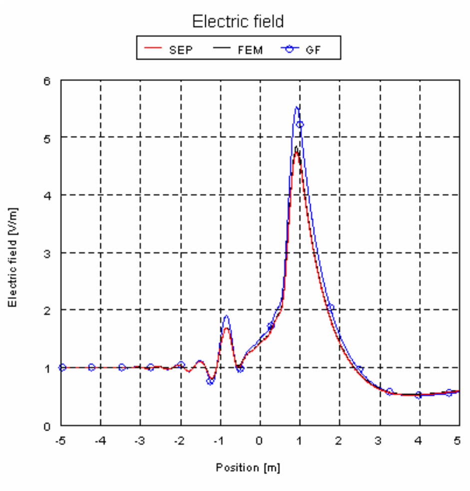
SEP: 16 sec (1 CPU), 50 MByte
edge length 0.2λ
VEP: 6 sec (1 CPU), 40 MByte
edge length 0.2λ
FEM: 16 sec (1 CPU), 150 MByte
edge length 0.2λ

Figure 3. Case 1 - Sphere with R = λ/4 with εr and μr = +/- 1.5

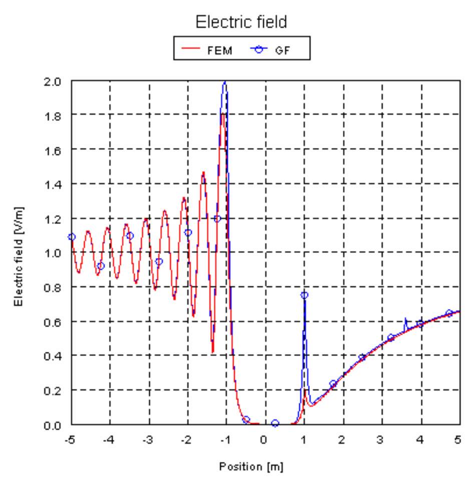
Figure 4 shows simulation results, optimal meshing details and resource requirements for case 2.

ε_r=+1,5 | ε_r = -1,5

μ_r=+1,5

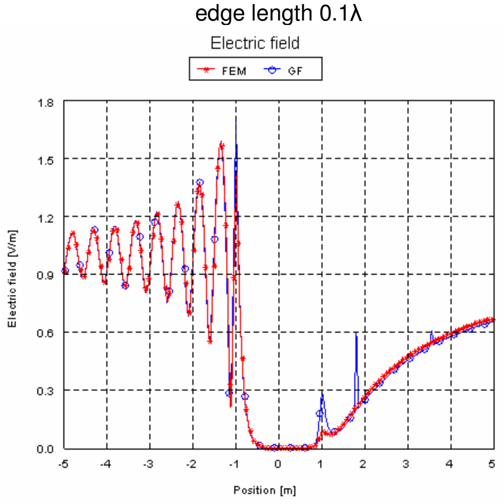


SEP: 16 sec (1 CPU), 40 MByte
edge length 0.2λ
FEM: 4 min (1 CPU), 600 MByte
edge length 0.1λ

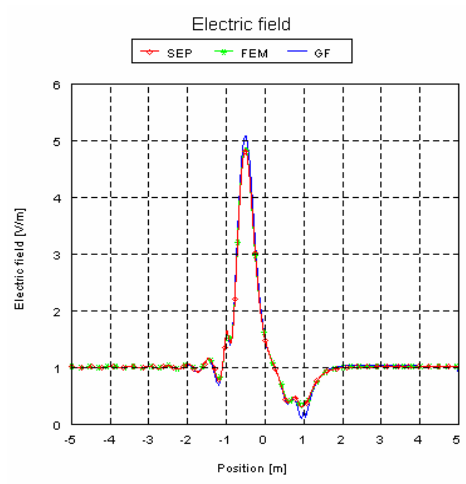


FEM: 6 min (1 CPU), 600 MByte
edge length 0.1λ

μ_r=-1,5



FEM: 10 sec (1 CPU), 50 MByte
edge length 0.2λ



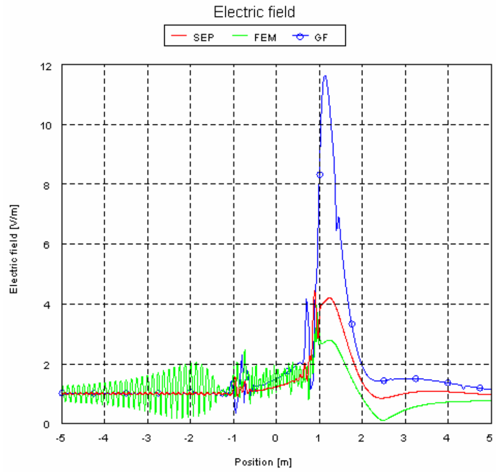
SEP: 16 sec (1 CPU), 50 MByte
edge length 0.2λ
FEM: 60 sec (1 CPU), 50 MByte
edge length 0.2λ

Figure 4. *Case* 2 - Sphere with R = 2λ with ε_r and μ_r = +/- 1.5

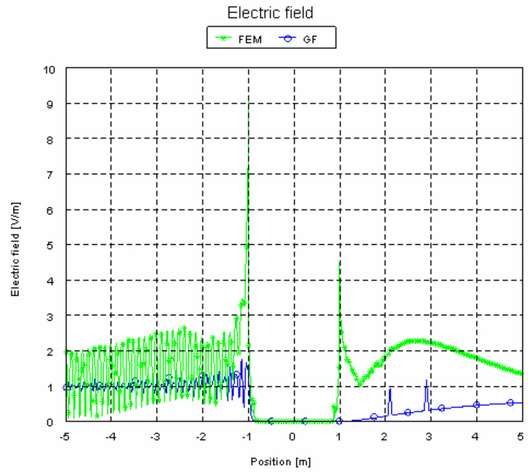
Figure 5 shows simulation results, optimal meshing details and resource requirements for case 3.

ε_r=+1,5 ε_r = -1,5

μ_r=+1,5

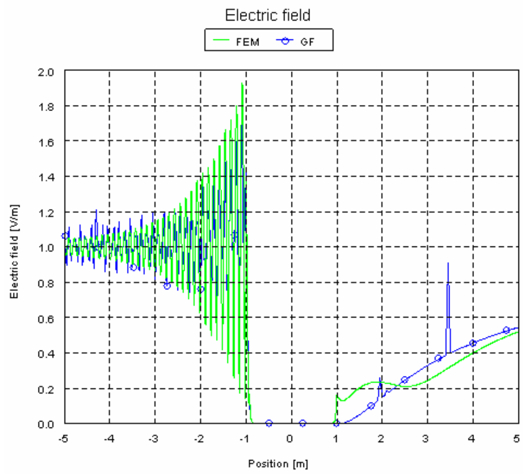


SEP: 8 min (1 CPU), 680 MByte
edge length 0.1λ
FEM: 4 min (1 CPU), 1.6 GByte
edge length 0.1λ

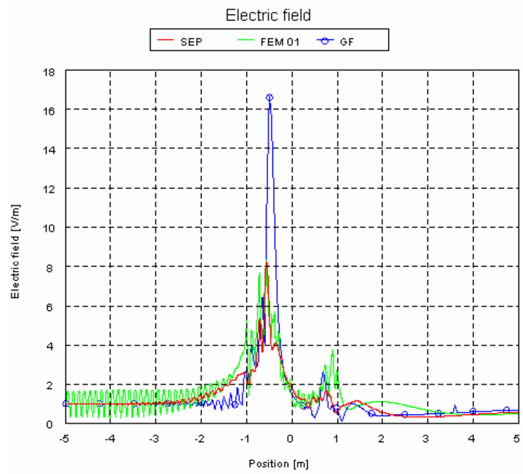


FEM: 16 sec (1 CPU), 50 MByte
edge length 0.2λ

μ_r=-1,5



FEM: 2 min (1 CPU), 580 MByte
edge length 0.1λ



SEP: 4 min (1 CPU), 680 MByte
edge length 0.1λ
FEM: 3 min (1 CPU), 1.6 GByte
edge length 0.1λ

Figure 5. Case 3 - Sphere with R = 8λ with εr and μr = +/- 1.5

Figure 6 shows simulation results, optimal meshing details and resource requirements for case 4.

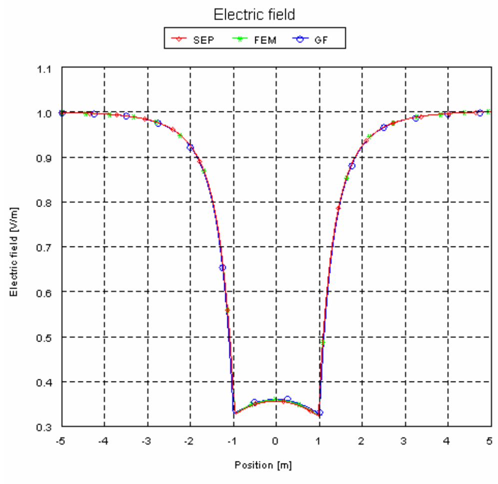
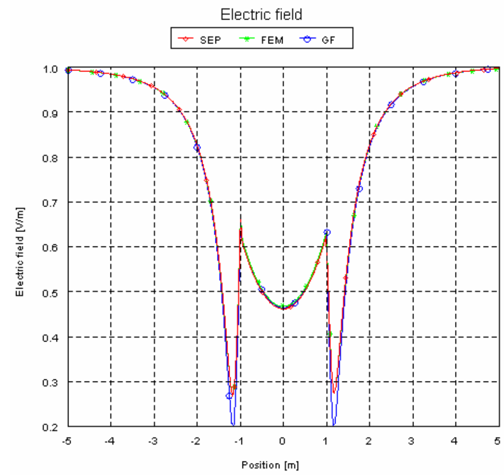
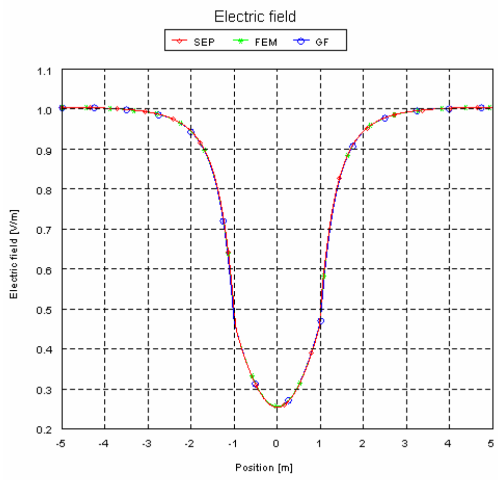
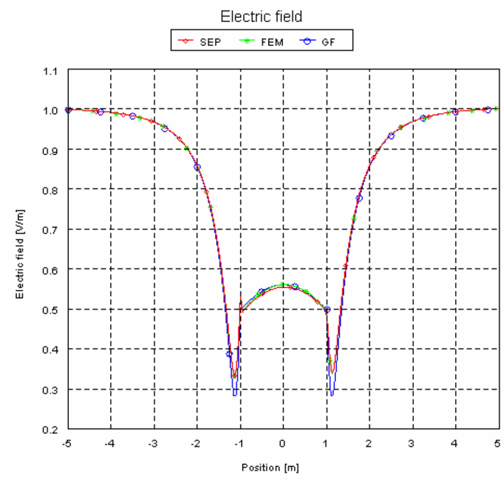
	ε_r=+1,5	ε_r = -1,5
μ_r=+1,5		
	SEP: 10 sec (1 CPU), 40 MByte edge length 0.2λ **FEM:** 12 sec (1 CPU), 50 MByte edge length 0.2λ	**SEP:** 10 sec (1 CPU), 40 MByte edge length 0.2λ **FEM:** 12 sec (1 CPU), 50 MByte edge length 0.2λ
μ_r=-1,5		
	SEP: 10 sec (1 CPU), 40 MByte edge length 0.2λ **FEM:** 12 sec (1 CPU), 50 MByte edge length 0.2λ	**SEP:** 10 sec (1 CPU), 40 MByte edge length 0.2λ **FEM:** 12 sec (1 CPU), 150 MByte edge length 0.2λ

Figure 6. *Case* 4: Sphere with R = $\lambda/2$ with ε_r and μ_r = +/- 8

Guidelines

From the sets of simulations using the sphere structure, the following negative index simulation guidelines can be assumed.

- Electrically large negative index regions deliver poor results for all techniques
- VEP should not generally be used, particularly for negative permittivity materials. The mesh is often not accurate for general geometries and the near-fields are not precise enough
- SEP or FEM can both be used for electrically smaller geometries. Here the SEP is better where possible due to smaller resource requirements but:

  - If $\varepsilon_r^* \mu_r < 0$ poor convergence/stability may be observed in the SEP, while better behaviour is found using the FEM approach. The use of the 'Direct Sparse Solver' for the FEM solution provides the most stable solution
  - If $\varepsilon_r^* \mu_r > 0$ the convergence/stability of the SEP is better and resource requirements are lower than for the FEM approach

- For electrically small to medium sized structures, high values of ε_r and μ_r can be handled accurately in both SEP and FEM implementations
- When using FEM, excessively fine meshing may reduce stability of the iterative solution. Refining of meshes beyond the convergence point will not therefore always improve solution accuracy.

A meta-material resonator structure

Technical description

A meta-material resonator structure from reference 2, as shown in Figure 7 was considered in FEKO using FEM/MoM. In the original paper, this structure was considered between two infinite PEC plates to force the TM mode. Finite plates were used in the FEKO model as shown.

The resonator was excited by placing a z-directed elementary dipole element at the centre of the structure.

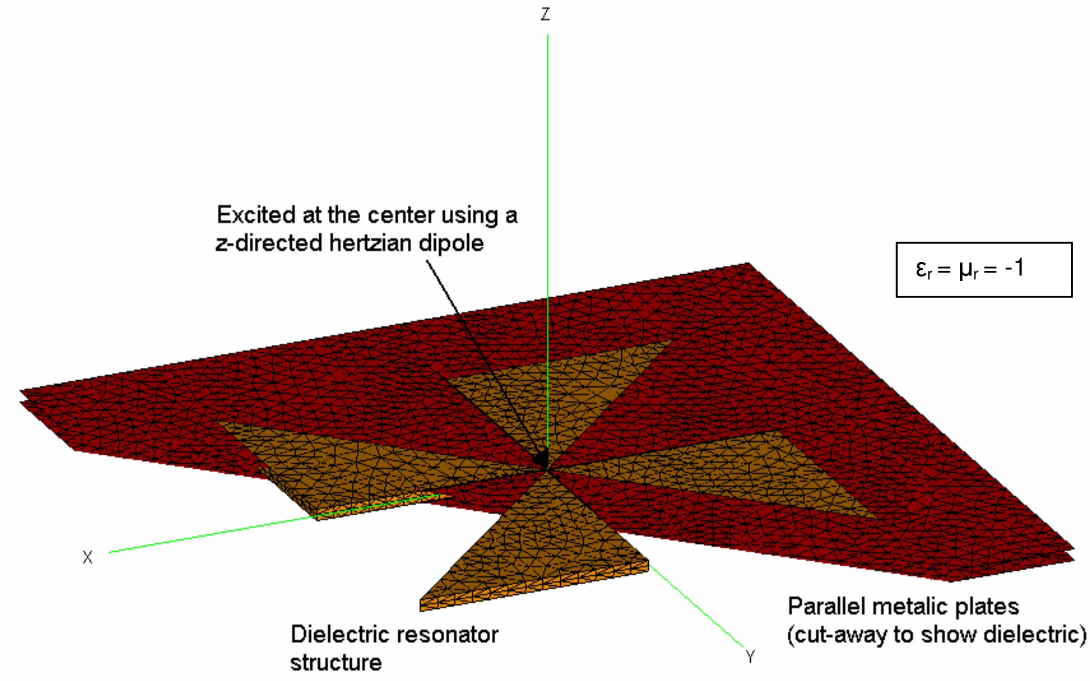


Figure 7. *Structure of the canonical meta-material resonator*

Simulation results

The magnitude of the near fields in the central plane of the structure at the resonant frequency are shown in Figure 1. The analysis was performed on an Intel Xeon processor, requiring 2.716GB peak memory and 15.63 minutes of processor time per frequency.